Automated Sailboat Navigation using Controllability-Aware Nonlinear Model Predictive Control under Stochastic Winds

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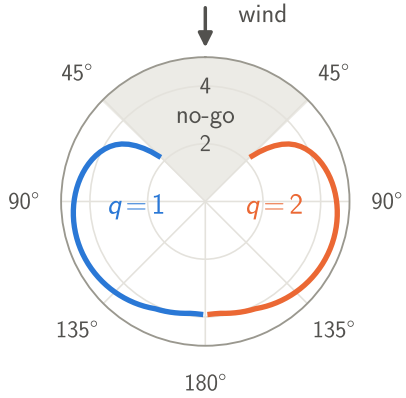
Motivation: sailing under stochastic wind

- Real wind is stochastic: gusts, direction shifts, fluctuating apparent wind speed
- Propulsion comes *from* the wind ⇒ uncertainty acts directly on the propulsive force
- Upwind no-go zone: the boat must *tack*, and its speed and heading authority collapse near it
- Line-of-sight and waypoint guidance assume a static, deterministic environment

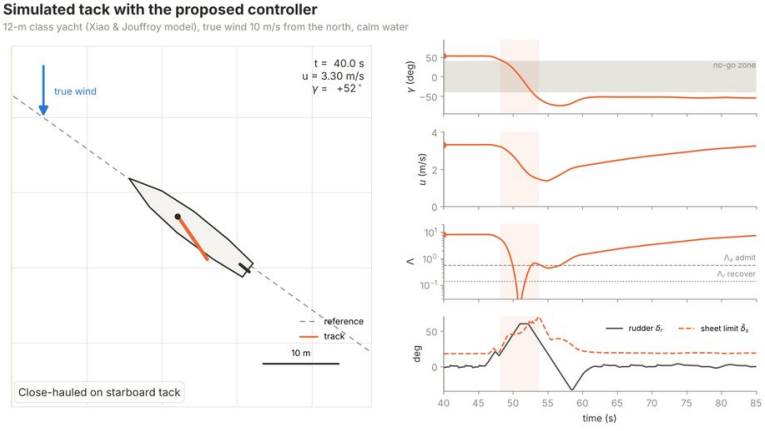
Question

How to plan *and* track time-efficient routes under stochastic wind, while keeping the boat maneuverable?



Polar speed $S(\beta)$ (m/s): steady speed at the angle β to the wind; $q = 1, 2$: wind on starboard, port

A tack, simulated



Proposed controller on a 200 m upwind leg, constant wind, 3× real time; shaded: tack window

State of the art and contributions

Stochastic route planning

- Dynamic programming (DP) over a random wind field, point-mass boat [Ferretti & Festa 2019]
- Semi-Lagrangian DP, probabilistic wind, tack switching [Miles & Vladimirsky 2022]
- No inertia, no actuator limits ⇒ references may be infeasible

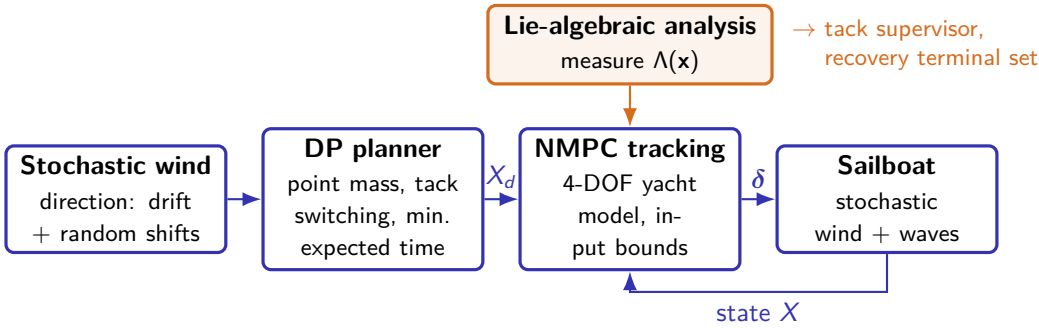
Nonlinear model predictive control (NMPC)

- Nonlinear dynamics, explicit constraints; few sailboat works [Liu et al. 2023; Tipsuwan et al. 2023]

Contributions

1. Stochastic wind-aware planner + NMPC tracking on a yacht model with 4 degrees of freedom (4-DOF: surge, sway, roll, yaw)
2. Lie-algebraic analysis of the control authority ⇒ online authority measure Λ
3. Λ in a tack supervisor + recovery terminal set: 24/24 races won in a shifting wind (baseline: 11/24)

Overall framework

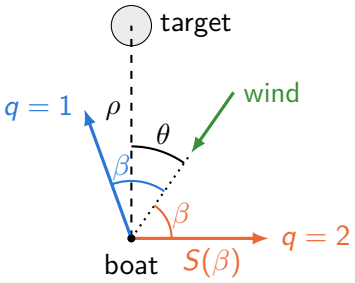


X_d: reference route of the point mass; $\boldsymbol{\delta} = (\delta_r, \bar{\delta}_s)$: rudder angle and sheet limit; X: measured state

Baseline: planner + NMPC tracking, input bounds only

Proposed: + a supervisor that admits, postpones or aborts the tacks with Λ, + a terminal set that keeps a recovery possible

Baseline planner: stochastic point mass



$$\begin{cases} d\rho = -S(\beta)\cos\big(\theta + (-1)^q\beta\big)\,dt \\ d\theta = \Big[\frac{S(\beta)}{\rho}\sin\big(\theta + (-1)^q\beta\big) + a_w\Big]dt + \sigma_w\,dW \end{cases}$$

- ρ: distance to the target; $\theta = \psi_w - \lambda$: wind direction ψ_w minus bearing λ of the line of sight (clockwise)
- $q \in \{1, 2\}$: tack; β: angle to the wind (course $\psi_w + (-1)^q\beta$); $S(\beta)$: polar speed
- a_w: drift of the wind direction; σ_w: shift intensity; W: Wiener process

Minimizes the expected time to the target; a tack costs a delay T_c [Miles & Vladimirsky 2022]

Limitation

No inertia, no actuator limits ⇒ the references may be infeasible

Why a controllability analysis?

Baseline NMPC: tracks the position, velocity and heading of the reference with bounded inputs $\boldsymbol{\delta}$. Nothing keeps the boat away from upwind states with low speed and low heading authority $\Rightarrow$ in gusts, turns and tacks may fail.

Control-affine system

$\dot{x} = f_0(x) + \sum_{i=1}^{m} f_i(x)\, u_i$

f_0: drift; f_i: input vector fields; u_i: inputs

Lie bracket: $[f, g] = \nabla g\, f - \nabla f\, g$ (∇: Jacobian)

- **LARC** (Lie Algebra Rank Condition): $f_0, \ldots, f_m$ and their iterated Lie brackets span the tangent space $T_x \Rightarrow$ *local accessibility* [Sussmann & Jurdjevic 1972]
- Controllability needs more (e.g. a recurrent drift); the sailboat drift is dissipative $\Rightarrow$ no controllability claim

In small time t [Boscain & Sigalotti 2019]

Motion along an input field f_i: $\sim t$

along a first-order bracket $[f_0, f_i]$: $\sim t^2$

Criterion used here

A direction no longer generated by the inputs and first-order brackets $\Rightarrow$ severe loss of maneuverability

Control-affine model with virtual inputs

Model non-affine in $(\delta_r, \bar{\delta}_s) \Rightarrow$ generalized forces τ as virtual inputs:

$$\begin{cases} \dot{\eta} = J(\eta)\,\nu \\ M\dot{\nu} = -C(\nu)\nu - D(\eta,\nu) - g(\eta) + \tau \\ \tau = \sum_{i\in\{x,y,\phi,\psi\}} k_i\,\tau_{\mathrm{aug},i} \end{cases}$$

$\eta = [x, y, \phi, \psi]^\top$: north, east positions, roll, heading; $\nu = [u, v, p, r]^\top$: surge, sway, roll, yaw rates

M: inertia, J: velocity transformation, C: Coriolis, D: damping, g: restoring

$\tau_{\mathrm{aug},i}$: actuator force along DOF i; $k_i \geq 0$: its effectiveness ($k_i = 0$: no actuation)

With $\mathbf{x} = [\eta^\top, \nu^\top]^\top \in \mathbb{R}^8$:

$$\dot{\mathbf{x}} = \mathbf{f}_0(\mathbf{x}) + \textstyle\sum_{i=1}^{4} \mathbf{f}_i\,\tau_{\mathrm{aug},i}$$

$\mathbf{f}_1 = k_x\mathbf{e}_5,\ \mathbf{f}_2 = k_y\mathbf{e}_6,\ \mathbf{f}_3 = k_\phi\mathbf{e}_7,\ \mathbf{f}_4 = k_\psi\mathbf{e}_8$ ($\mathbf{e}_j$: j-th unit vector)

Virtual vs. physical inputs

$(\delta_r, \bar{\delta}_s)$ act only through τ: rank loss (virtual) $\Rightarrow$ rank loss (real boat), not conversely: necessary conditions

First-order Lie brackets and rank

$\mathbf{f}_i$ constant $\Rightarrow \mathbf{f}_{4+i} \triangleq [\mathbf{f}_0, \mathbf{f}_i] = -\nabla \mathbf{f}_0 \, \mathbf{f}_i, \quad [\mathbf{f}_i, \mathbf{f}_j] = 0$

$\mathcal{L}_1(\mathbf{x}) = [\mathbf{f}_1, \ldots, \mathbf{f}_8]$: inputs + first-order brackets

$$\mathcal{L}_1 = \begin{bmatrix} 0 & E(\mathbf{x}) \\ K & G(\mathbf{x}) \end{bmatrix}, \quad K = \operatorname{diag}(k_x, k_y, k_\phi, k_\psi)$$

E_1

$$E = \begin{bmatrix} -k_x \cos\psi & k_y \sin\psi \cos\phi & 0 & 0 \\ -k_x \sin\psi & -k_y \cos\phi \cos\psi & 0 & 0 \\ 0 & 0 & -k_\phi & 0 \\ 0 & 0 & 0 & -k_\psi \cos\phi \end{bmatrix}$$

E_2

E: action on positions and angles; G: on the rates (not needed for $\det \mathcal{L}_1$)

Rank condition

$\det \mathcal{L}_1 = \det E_1 \det E_2 \det K$

$\det E_1 = k_x k_y \cos\phi$
$\det E_2 = k_\phi k_\psi \cos\phi$
$\det K = k_x k_y k_\phi k_\psi$

$\Rightarrow \det \mathcal{L}_1 = k_x^2 k_y^2 k_\phi^2 k_\psi^2 \cos^2\phi$

- $|\phi| < \pi/2$: full rank $\iff$ all $k_i \neq 0$
- $k_x = 0$, $v = r = 0$: rank 6 at first order, 8 with second-order brackets: accessible, but slowly

Surge-degenerate set

$\mathcal{L}_0(\mathbf{x}) = [\mathbf{f}_0, \mathbf{f}_1, \ldots, \mathbf{f}_8] \in \mathbb{R}^{8\times 9}$: its 5th row gathers every first-order contribution to the surge acceleration $\dot{u}$ (drift, sail, sway–yaw coupling)

5th row $\equiv 0 \iff$

$$k_x = 0 \quad \text{(C1)}$$
$$-\dot{Y}_v r v + m r v + \sin\alpha_k - \cos\alpha_h - \cos\alpha_k = 0 \quad \text{(C2)}$$
$$k_y(\dot{Y}_v - m)\, r = 0 \quad \text{(C3)}$$
$$k_\psi(\dot{Y}_v - m)\, v = 0 \quad \text{(C4)}$$

m: mass; $\dot{X}_u$, $\dot{Y}_v$: surge and sway added mass; α_k, α_h: angles of attack of the keel and of the hull

- $k_y, k_\psi \neq 0$ and $\dot{Y}_v \neq m$: (C3)–(C4) $\iff v = r = 0$ (implied, not assumed)
- (C2) at $v = r = 0$: degraded region

$$p \;\geq\; u\,(\cos\phi + 1)$$

Why surge?

Forward speed $\Rightarrow$ rudder effectiveness $\Rightarrow$ every other maneuver: losing the surge channel is the most critical degradation

Propulsion: when is $k_x > 0$?

Modeling assumption: $k_x = \kappa(\tau_x - F_{rh})$, $\kappa(\xi) = 0$ if $\xi \leq 0$, $\kappa(\xi) > 0$ if $\xi > 0$

- Surge force: $\tau_x = L_s \sin\alpha_{aw} - D_s \cos\alpha_{aw} - D_r$
- Sail lift and drag: $L_s, D_s = \frac{1}{2}\varrho_a A_s V_{aw}^2 \, C_{L_s,D_s}(\alpha_s)$
- Rudder drag: $D_r = \frac{1}{2}\varrho_w A_r V_{ar}^2 \, C_{D_r}(\alpha_r)$
- $F_{rh}(V_{ah})$: hull resistance at the hull speed V_{ah} (polynomial fit)

ϱ_a, ϱ_w: air and water densities; A_s, A_r: sail and rudder areas

V_{aw}, α_{aw}: apparent wind speed and angle at the sail; V_{ar}, α_{ar}: inflow speed and angle at the rudder

$\alpha_s = \alpha_{aw} - \delta_s$, $\alpha_r = \alpha_{ar} - \delta_r$: angles of attack ($\delta_s$: sail angle)

Surge channel actuated

$k_x > 0 \iff \tau_x > F_{rh}(V_{ah})$: sail thrust minus rudder drag exceeds the hull resistance

From (C1)–(C4) to NMPC constraints

The surge-degenerate set needs (C1)–(C4) together $\Rightarrow$ violating any one of them keeps the boat out of it, so each constraint below excludes the set on its own:

Constraints implied by (C1)–(C4)

(C1)	$k_x = 0$	$\rightarrow$	$\tau_x > F_{rh}$	propulsion
(C2)	$p \geq u(\cos\phi + 1)$	$\rightarrow$	$p < u(\cos\phi + 1)$	roll rate bounded by surge
(C3)–(C4)	$v = r = 0$	$\rightarrow$	$\lvert v\rvert \geq \epsilon_v,\ \lvert r\rvert \geq \epsilon_r$	margins on sway and yaw rates

- $\epsilon_v = 0.02\,\mathrm{m/s}$, $\epsilon_r = 0.005\,\mathrm{rad/s}$: small margins
- P (pointwise constraints): the baseline NMPC + these four constraints at every step of the horizon, soft (penalized slack), relaxed while the boat tacks

From $\det \mathcal{L}_1$ to an online authority measure

Effectiveness k_i = width of the force that the actuators can produce along i:

$$\begin{aligned} k_i(\mathbf{x}) &= \max_{\boldsymbol{\delta}} \tau_i(\mathbf{x}, \boldsymbol{\delta}) - \min_{\boldsymbol{\delta}} \tau_i(\mathbf{x}, \boldsymbol{\delta}) \\ &= q_s \, \Delta C_i^s(\alpha_{aw}) + q_r \, \Delta C_i^r(\alpha_{ar}) \end{aligned}$$

$q_s = \frac{1}{2}\varrho_a A_s V_{aw}^2$, $q_r = \frac{1}{2}\varrho_w A_r V_{ar}^2$; $\Delta C_i^{s,r}$: widths of the sail and rudder coefficients when $\bar{\delta}_s$ and δ_r sweep their ranges

Authority measure

$$\Lambda(\mathbf{x}) = \sqrt{\frac{\det \mathcal{L}_1(\mathbf{x})}{\det \mathcal{L}_1(\mathbf{x}_{\mathrm{ref}})}} = \frac{\cos\phi}{\cos\phi_{\mathrm{ref}}} \prod_i \frac{k_i(\mathbf{x})}{k_i(\mathbf{x}_{\mathrm{ref}})}$$

$\mathbf{x}_{\mathrm{ref}}$: steady state at 40° to the wind (2.3 m/s), where $\Lambda = 1$; $\Lambda = 0 \iff$ first-order rank lost

Choices

- Width, not $\kappa(\tau_x - F_{rh})$: $\tau_x = F_{rh}$ at any steady state
- Quasi-static: p, r removed (roll adds up to 4.6 m/s of apparent wind)
- Authority $\neq$ propulsion: keep $\tau_x \geq F_{rh}$

γ: angle between heading and wind

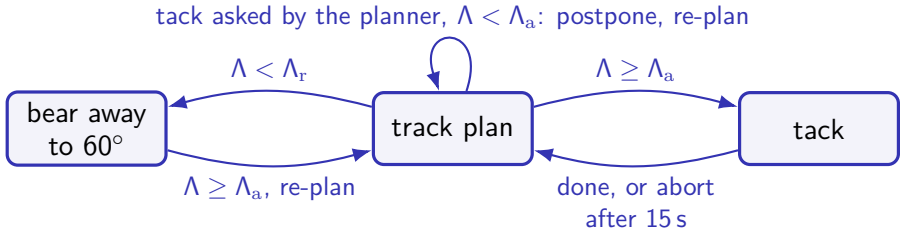
speed (m/s) at $\lvert\gamma\rvert =$	30°	40°	50°
tack, $\Lambda_{\mathrm{a}} = 0.56$	2.4	1.9	1.7
recover, $\Lambda_{\mathrm{r}} = 0.13$	1.6	1.2	1.0

Proposed controller

1. Recovery terminal set, on the last predicted state $\mathbf{x}_N$

$|\gamma_N| \geq 40^\circ$ (no-go zone), $u_N \geq 1.5\,\mathrm{m/s}$ (rudder authority), $\tau_x \geq F_{rh}$ (propulsion)
P imposes the four constraints of (C1)–(C4) at every step. Here the margins on p, v, r stay at every step, and $\tau_x > F_{rh}$ moves to $\mathbf{x}_N$: the boat may ease the sheet and bear away within the 2 s horizon

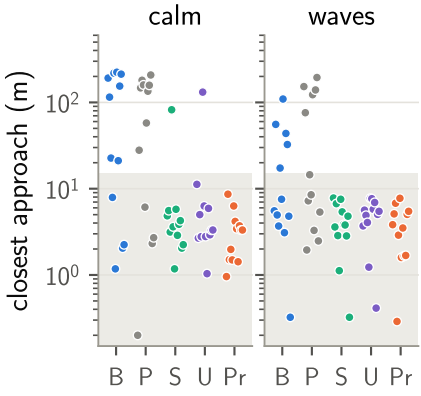
2. Tack supervisor on Λ



Λ_a: admission threshold, Λ_r: recovery threshold, equal to 2 m/s and 1.2 m/s at 40° from the wind. Bearing away: heading 60° from the wind, no position or speed term in the cost.

Simulation setup

- **Boat**: Xiao–Jouffroy 4-DOF model of a 12-m yacht ($m = 25.9\,\mathrm{t}$, sail area $A_s = 170\,\mathrm{m}^2$), lift, drag and hull-resistance curves digitized from their paper; true wind speed $10\,\mathrm{m/s}$
- **Planner**: Miles–Vladimirsky scheme, tack delay $T_c = 12\,\mathrm{s}$, polar computed from the boat model
- **NMPC**: sampling time $\Delta t = 0.2\,\mathrm{s}$, horizon $N = 10$ steps ($2\,\mathrm{s}$); inputs: rudder angle $|\delta_r| \leq 60^\circ$ and sheet limit $\bar{\delta}_s \in [2^\circ, 90^\circ]$; sail angle $\delta_s = \mathrm{sat}_{\pm\bar{\delta}_s}(\alpha_{aw})$ (luffing model); CasADi / IPOPT
- **Races**: 250 m upwind; wind direction drifting at $a_w = 0.6^\circ/\mathrm{s}$ with shifts $\sigma_w = 0.06\,\mathrm{rad}/\sqrt{\mathrm{s}}$; 12 wind realizations, in calm water and with waves (10 % of the nominal surge and sway forces, 100 % of the roll and yaw moments): 24 races
- **Race won**: the boat comes within 15 m of the goal
- **Plan**: the point mass that follows the planner's policy in the same wind; its arrival time is the reference for the delays



Closest approach to the goal; shaded: race won

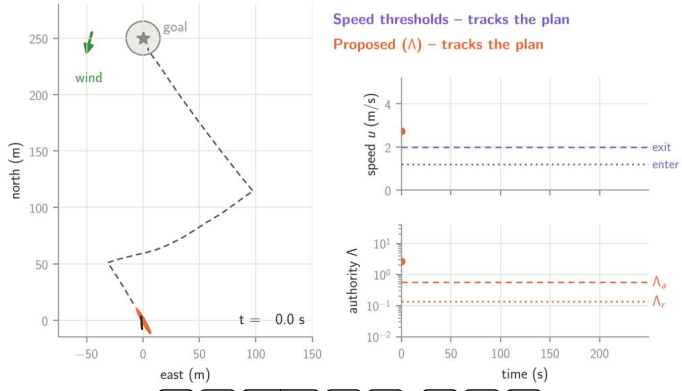
Monte Carlo: 24 races

	Won	Worst delay
Baseline (B)	11/24	13 s
Pointwise constraints (P)	11/24	15 s
Proposed (Pr)	24/24	9 s
Ablations of Pr		
Speed thresholds, not Λ (U)	23/24	126 s
No recovery set (S)	23/24	32 s
$\Lambda_N \geq 0.25$ in terminal set	24/24	22 s

Delay: arrival − planned arrival, worst of the races won (B, P: easier races only)

- Supervisor: most of the gain; recovery set: one more race, shorter worst delay
- U (tack at 2 m/s, recover below 1.2 m/s): blocked recoveries in 3 calm races

Why Λ rather than speed thresholds? Race 1



Same wind, 8× real time; after the first tack both boats are slow ($\approx 1.2\,\mathrm{m/s}$)

Speed rule: recovery, with no speed term in its cost: below 2 m/s, 173 s in recovery

Λ: slow but off the wind, $\Lambda > \Lambda_r$: no recovery, finishes 128 s earlier

Conclusion and perspectives

Take-home

- Stochastic DP planner + NMPC on the Xiao–Jouffroy yacht
- Lie analysis: $\sqrt{\det \mathcal{L}_1} = k_x k_y k_\phi k_\psi \cos\phi$ $\Rightarrow$ online authority measure Λ
- P, the constraints from (C1)–(C4) at every step: no gain (11/24, as the baseline)
- Λ in a tack supervisor + recovery terminal set: 24/24 races, at most 9 s behind the plan

Limitations and next steps

- Thresholds calibrated on speed thresholds: derive them from the analysis
- Recovery mode without a speed objective
- The 2 s horizon is too short to plan a tack: an optimal control problem over the whole 23 s tack works, but needs 0.3 s per step
- 12 races per condition; experiments on a real sailboat

Thank you!

Questions?